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HEAT TRANSFER IN SUPERSONIC SEPARATED FLOW OVER A TWO-DIMENSIONAL BACKWARD-FACING STEP

P. J. BAKER and B. W. MARTIN
Department of Mechanical Engineering, Imperial College of Science and Technology, London

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Abstract—Measurements of heat transfer are reported in the entry enlargement region of a supersonic parallel diffuser, where flow separation occurs. Very high heat transfer is observed near re-attachment with presumed transitional separation. After re-attachment following laminar separation, the heat transfer is less with supersonic flow than with the subsonic flow resulting from a normal shock at re-attachment.

NOMENCLATURE

A,	cross-sectional area;
h,	local heat-transfer coefficient, based on local heat flux and difference between surface and gas-recovery temperatures;
k,	thermal conductivity;
l,	length for boundary-layer development before separation;
m,	total mass flow rate of gas;
M,	Mach number;
Nu,	Nusselt number, hl/k;
Pr,	Prandtl number, ν/α;
r,	temperature recovery factor;
Re,	Reynolds number $ml/A\mu$;
T,	temperature.

Greek symbols

α,	thermal diffusivity;
γ,	ratio of specific heats;
μ,	absolute viscosity;
ν,	kinematic viscosity.

Subscripts

a,	value at re-attachment;
d,	value at edge of free shear layer;
l,	value before separation (at nozzle outlet);
m,	maximum value;
o,	stagnation condition;
r,	recovery value;
s,	main stream value immediately downstream of shock;
x,	at flow station downstream of step.

Superscript

$*$,	value at nozzle throat.

INTRODUCTION

MEASUREMENTS of heat transfer in supersonic separated flow over a backward-facing step under laminar conditions have been reported in several papers. Brinich [1] and Naysmith [2] used a cone-cylinder model in a wind tunnel. Respectively, the steps were created by (1) a forward-facing wedge mounted on the model (2) a cylinder whose diameter was less than that of the base of the cone. Laminar heat transfer measurements were made in a shock tube by Rom and Seginer [3] over a two-dimensional backward-facing step machined in a flat plate behind a sharp leading edge. Table 1 gives details of step heights and flow conditions prior to separation.

Although heat transfer at the beginning of separation was as little as 10 per cent of that before separation in references [2] and [3], further downstream it rose, often rapidly, to a peak value at or near re-attachment. Naysmith's [2] maximum was 3·05 times the pre-separation heat transfer, while Rom and Seginer [3] found that $(h_x/h_l)_m$ rose from 1·6 to 4·6 when Re_l increased from 2×10^4 to 1×10^5, followed by a jump in the maximum to 7·1 at $Re_l = 2 \times 10^5$. This may indicate a transition to turbulence just before or during separation. Brinich [1] found a comparable increase in heat transfer downstream of the wedge, with $(h_x/h_l)_m \simeq 7{\cdot}5$, but the instrumentation was too sparse to show either the increase during separation or the heat transfer after re-attachment.

Table 1

Author	Step height (in)	Mach number M_l	Reynolds number $Re_l/10^3$
Brinich	0·08	3·1	10000
Naysmith	1·0	1·7	2000
Rom and Seginer	0·06	1·5–2·5	2–200

Brinich [1] and Naysmith [2] also studied turbulent separation; the latter used a forward-facing wedge on the bottom wall of a wind tunnel. Other wind tunnel measurements are reported by Thomann [4], Gadd *et al.* [5] and Charwat *et al.* [6] for a backward-facing step on an otherwise uniform flat surface. Step heights ranged from 0·08 in to 1 in, for M_l from 1·8 to 4, and Re_l from $1{\cdot}5 \times 10^6$ to 10^7. Again the heat transfer increased in the separated region; Charwat *et al.* [6] suggest a slight Mach number effect. Values of $(h_x/h_l)_m$ are generally much lower than in laminar flow, though comparisons should have regard to the increase in h_l for turbulent separation. The highest recorded value of $(h_x/h_l)_m$, by Gadd *et al.* [5] at $Re_l = 6 \times 10^6$ and $M_l = 2{\cdot}44$, was 1·35. Downstream of re-attachment the heat transfer for both laminar and turbulent separation generally decreased to approach h_l almost asymptotically in some cases.

The present paper describes previously unreported measurements in the entry region of a long supersonic parallel-sided diffuser. Separation occurs because of a sudden enlargement in flow area where the supersonic jet generated by an upstream nozzle enters the diffuser, to which the nozzle is connected, thus forming a backward-facing step. The nozzle is supplied with high-pressure air and the diffuser discharges to atmosphere.

The equipment was primarily designed for a wider investigation involving schlieren observation of the flow patterns, the associated wall static pressure distributions and local heat-transfer measurements throughout the diffuser, and was instrumented accordingly. A full description may be found in a previous paper by the authors [7], and below are given only those features relevant to the results presented herein. These appear to support a possibility hinted at by Naysmith [2], and strengthened by the subsequent findings of Rom and Seginer [3], which may be stated as follows. When Re_l is such that transition from laminar to turbulent flow occurs along the separated shear layer, the peak heat transfer in the re-attachment zone is much higher than when the separated flow is wholly laminar or wholly turbulent.

An advantage of using a supersonic diffuser with an entry enlargement is that, by control of the nozzle stagnation pressure, the separated region may be investigated not only when an oblique shock is formed in the re-attachment region (such as occurs in the other models investigated) but also when this is replaced by a single normal shock which bifurcates near the diffuser wall. The downstream flow is then (or soon becomes) wholly subsonic, in contrast to the still supersonic stream following an oblique shock. The resulting difference in Mach number between these two shock systems downstream of re-attachment is quite large, and while present evidence is not conclusive, it suggests that, after re-attachment, heat transfer is less with supersonic flow than with subsonic flow.

EXPERIMENTAL APPARATUS AND PROCEDURE

The two-dimensional model used consists of a convergent–divergent nozzle discharging into a parallel-sided diffuser. Both are of rectangular cross-section. The nozzle has a throat section 4 in by 0·75 in and an outlet section 4 in by 0·9 in. The divergence is 0·976 in long and has a uniform taper of 5°. The nozzle area ratio A_t/A^* of 1·2 corresponds to a design outlet Mach number of 1·54 at an overall pressure ratio of 3·86. The actual M_t was determined from subsidiary experiments using a 16·5° wedge-shaped probe placed in the nozzle exit plane to create an attached oblique shock whose wave angle was measured. Confirmation was provided by independent measurements of the stagnation pressure loss through the nozzle.

The diffuser is 26 in long and its cross-section is 4 in by 2·85 in. It is connected to the nozzle so that (a) their axes of symmetry coincide (b) the flow cross-section has a uniform width of 4 in. Although the flow is nominally two-dimensional it is recognized that even this width may be insufficient to avoid the development of cellular flow patterns downstream of re-attachment. The simplified half-sectional views of the nozzle divergence and the diffuser entry in the upper panels of Figs. 1 and 2, which are nevertheless drawn to scale, show the backward-facing step of height 0·975 in, and the separated region. The shock systems are reproduced from measurements taken from schlieren photographs of the flow. Quoted Mach numbers have been calculated from measured flow deflection and wave angles.

The upper horizontal surface of the diffuser is a mild steel plate with provision for static

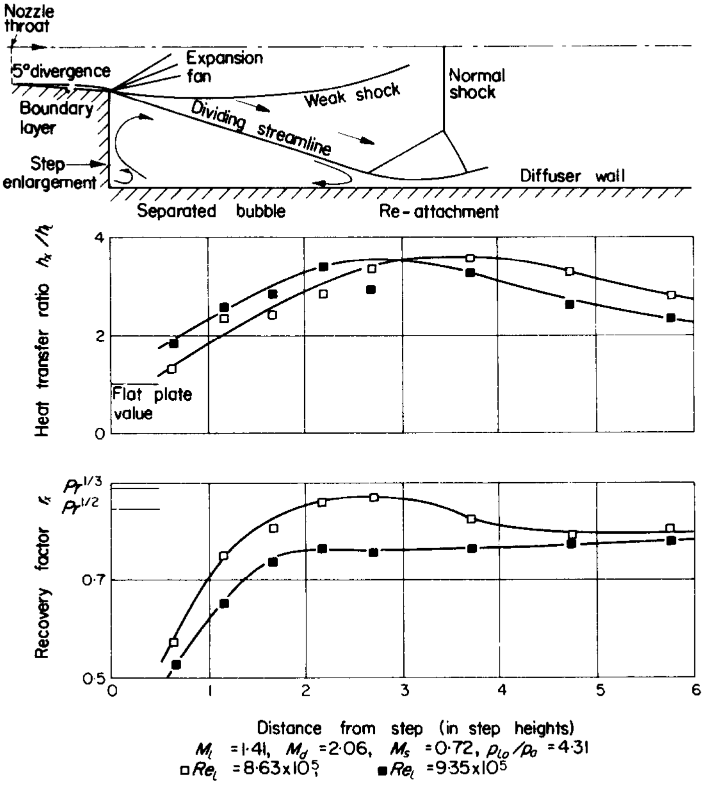

FIG. 1. Flow patterns, relative heat-transfer coefficients and temperature-recovery factors with normal shock regime.

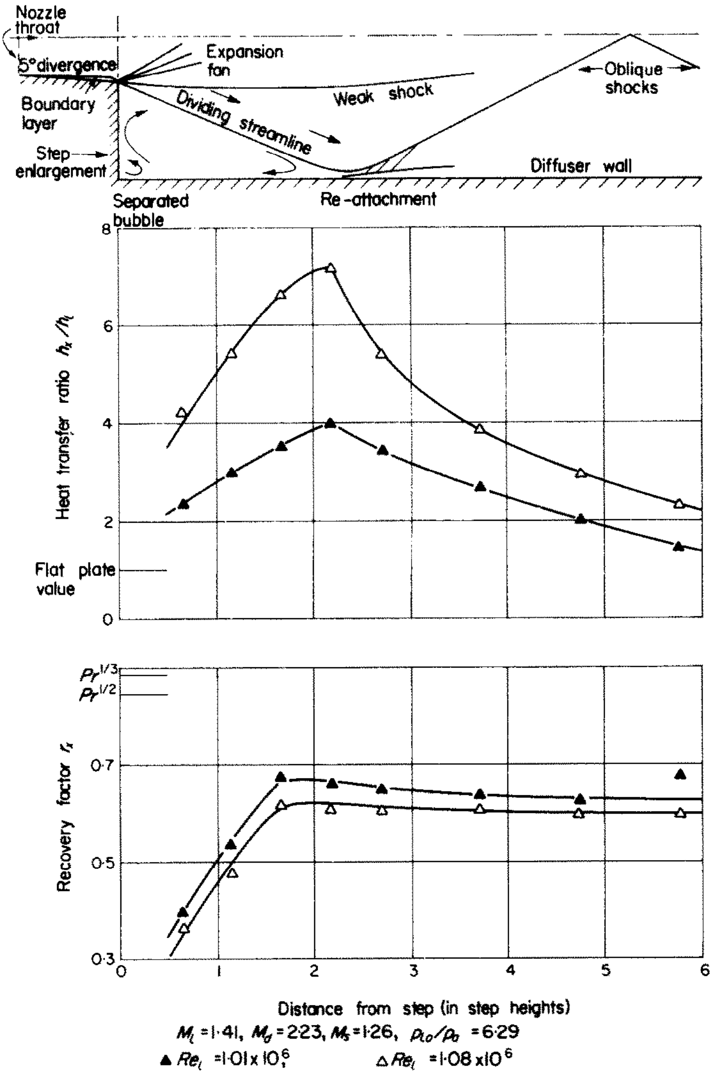

FIG. 2. Flow patterns, relative heat-transfer coefficients and temperature-recovery factors with oblique shock regime.

pressure taps along its centre line at $\frac{1}{2}$-in pitch. Heat transfer to the diffuser wall is measured by a number of copper–plastic units, of sandwich construction, set flush in the lower horizontal plastic surface along its centre line, also at $\frac{1}{2}$-in pitch, and all communicating with an underslung water jacket. Each copper–plastic stud consists of a polypropylene disk $\frac{1}{4}$-in diameter and $\frac{1}{32}$ in thick, stuck by impact adhesive between two copper elements each $\frac{1}{4}$-in diameter and $\frac{1}{4}$ in long, as illustrated in reference [7].

Copper–constantan thermocouples were used to measure the temperatures of the copper elements, with the hot junctions embedded halfway along the central axis of each copper element. If the temperature gradients in the copper elements are neglected in comparison with that across the plastic disk, the heat flux through the measuring unit can be determined from the temperature difference across the plastic disk, its thickness and thermal conductivity.

The corresponding value of h is based on the difference between the temperature of the copper element set flush in the diffuser wall and the temperature of the same element at zero heat flux, i.e. the recovery temperature T_r. The pitch of the measuring units which, as Naysmith [2] points out, should be made small to improve the identification of peaks in the heat transfer, was based in this case on calculations aimed at minimizing heat transfer between adjacent measuring units by conduction across the plastic wall of the diffuser. These considerations are most nearly reconciled by using a large number of small diameter studs.

Measurements were made at a gas stagnation temperature T_{lo} of 373°K (100°C), which remained sensibly constant along the diffuser. Acceptable temperature difference across the heat-transfer measuring studs were created by circulating water through the cooling jacket at 275°K (2°C) and at a sufficiently high rate to prevent any noticeable rise in temperature across the jacket. The overall thickness of the plastic disk and adhesive was determined to an accuracy within 0·6 per cent and the thermal conductivity of polypropylene to within 5 per cent. Including errors of up to 6 per cent in measuring (a) the temperature difference across the plastic disk (b) the difference between the surface temperature and the gas recovery temperature, the overall error in measuring h should therefore not exceed 12 per cent. The nozzle entry pressure was measured to within 1 per cent.

Because of the small nozzle dimensions, it was impracticable to measure directly either the boundary-layer thickness or the heat transfer at the nozzle exit. Instead, predicted values of h_l for attached laminar flow over a flat plate were derived from

$$Nu_l = 0{\cdot}332\, Re_l^{\frac{1}{2}}\, Pr^{\frac{1}{3}} \tag{1}$$

Re_l was obtained from the mass flow through the nozzle with the distance from throat to outlet as characteristic dimension. In calculating heat transfer, property values were introduced at a mean temperature. Reasons for the assumption of laminar flow are given below.

The temperature recovery factor r_x, which is related to T_{rx} by the equation

$$r_x = \frac{T_{rx}}{T_{ol}}\left[1 + \frac{2}{(\gamma - 1)\,M_l^2}\right] - \frac{2}{(\gamma - 1)\,M_l^2}, \tag{2}$$

is based on measured values of T_{ol}, T_{rx} and M_l. The pre-separation condition is a conventional reference (a) when evaluating r_x (b) with which to compare h_x, and has been used solely to accord with existing practice and thus to facilitate comparison with other work. In the authors' view, such reference often leads to unrealistic conclusions, at least when separation is caused by a step. This point is elaborated below.

RESULTS AND DISCUSSION

The nozzle inlet stagnation pressures used correspond to values of $Re_l/10^5$ of 8·63, 9·35, 10·1 and 10·8. Throughout the tests M_l was 1·41, corresponding to an overall nozzle isentropic efficiency of 0·96. For a given nozzle-diffuser system discharging against a constant pressure, the authors [7] have shown the inlet stagnation pressure to have a critical value (straddled by the present tests), above which a normal shock near re-attachment is superseded by an oblique shock system. As the pressure is further increased this spreads in lazytongs fashion downstream. Re_l values of $8{\cdot}63 \times 10^5$ and $9{\cdot}35 \times 10^5$ are associated with the normal shock regime, as shown in Fig. 1, while $1{\cdot}01 \times 10^6$ and $1{\cdot}08 \times 10^6$ correspond to the oblique shock regime in Fig. 2.

The change from a normal shock to an oblique shock regime has the following consequences:

(1) The Mach number of the quasi-inviscid flow at the edge of the shear layer M_d is increased from 2·06 to 2·23 because of the slightly greater Prandtl–Meyer expansion at separation. This derives from the increased main stream static pressure ahead of the shock system.

(2) The ratio of nozzle outlet stagnation pressure to maximum wall static pressure at re-attachment p_{to}/p_a is increased from 4·31 to 6·29.

(3) Immediately downstream of the shock wave the main stream Mach number M_s just outside the boundary-layer is increased from a subsonic value of 0·72 to a supersonic value of 1·26.

(4) Values of M_d, M_s and p_{to}/p_a referred to in (1), (2) and (3) for either shock regime are not appreciably influenced by the quoted change in Re_t within that regime.

As already stated, the flow before separation was not determined from measurements of the boundary-layer thickness. The high nozzle isentropic efficiency and the favourable pressure gradient during expansion suggest that laminar flow was preserved above the normal transition Reynolds number of 5×10^5 for boundary-layer flow along a flat surface with zero pressure gradient.

Further indirect evidence of laminar flow is provided in Fig. 1. Values in the region of 3·6 for $(h_x/h_t)_m$ are (a) comparable with those achieved in laminar flow by Naysmith [2] at $Re_t = 2 \times 10^6$ and Rom and Seginer [3] at $Re_t = 8 \times 10^4$ but (b) considerably higher than the maximum of 1·35 for turbulent flow at $Re_t = 6 \times 10^6$ reported by Gadd *et al.* [5].

The heat-transfer measurements for $Re_t = 1·01 \times 10^6$ in Fig. 2 show that in the separated region the rate of increase of h_x/h_t with Re_t in Fig. 1 is maintained, despite the change in shock regime and the slight increase in M_d. But the position of $(h_x/h_t)_m = 3·97$ lies further upstream.

In Fig. 2, when $Re_t = 1·08 \times 10^6$, values of h_x/h_t in the separation and re-attachment zones are very much larger than those predicted by extrapolating the rate of increase of h_x/h_t at lower Re_t. This sudden rise in heat transfer is not readily attributable to a change in Mach number, whose values are determined by the shock system and, as already stated, are virtually independent of the increase in Re_t from $1·01 \times 10^6$ to $1·08 \times 10^6$. A more likely explanation of the increased heat transfer is the onset of transition to turbulence during separation. Sterret and Holloway [8] note a similar effect with forward-facing steps; when transition is complete, heat transfer in the re-attachment zone falls to a lower value. This is supported by the turbulent flow measurements of Brinich [1], Thomann [4], Gadd *et al.* [5] and Charwat *et al.* [6]. Limitations on the air supply available unfortunately precluded investigation at Re_t above the cited range.

It is probably coincidental that the value of $(h_x/h_t)_m = 7·19$ for $Re_t = 1·08 \times 10^6$, where $h_{xm} = 394$ CHU/h ft^2 degC, agrees closely with the peak measurement of 7·1 by Rom and Seginer [3] at $Re_t = 2 \times 10^5$ using a much smaller step. The comparison nevertheless helps to confirm that so large a multiplication of heat transfer is possible in supersonic re-attaching flow. If further substantiated, it would have an important bearing on the design of such devices as the diffusers described by the authors [7].

Reference to Fig. 1 shows that in subsonic flow downstream of the normal shock the effect of Re_t on h_x/h_t is reversed, compared with that during separation. Moreover, the heat transfer approaches the flat plate value much more slowly than in Fig. 2, where with supersonic flow downstream of re-attachment, the improvement in heat transfer with Re_t is preserved. At five or six step heights from the step, h_x/h_t is less at $Re_t = 1·01 \times 10^6$ and $M_s = 1·26$ (Fig. 2), than at $Re_t = 8·63 \times 10^5$ and $M_s = 0·72$ (Fig. 1).

Reduced heat transfer at the higher Mach number, notwithstanding the larger Re_t, is probably connected with increased laminar boundary-layer stability. In the triangular region bounded by two adjacent oblique shocks and the diffuser wall, the overall static pressure gradient is favourable. By contrast, following a normal shock, the generally adverse pressure gradient and falling subsonic Mach number, as the mainstream spreads towards the walls, encourages the early formation of a turbulent boundary-layer with higher heat transfer.

The plateaux in h_x/h_l in Fig. 1, may be contrasted with the sharp peaks in Fig. 2, together with their location relative to the position of re-attachment determined by flow observation. However, the pitch of the measuring studs is insufficiently fine to ensure that these positions are accurately compared, or that intermediate values of local heat transfer are truly represented by the curves drawn. For the same reason, lack of evidence of h_x/h_l below unity immediately after separation, as obtained by Naysmith [2] and Rom and Seginer [3], should not be overstressed.

The reference heat transfer h_l is that just before separation. This corresponds to the growth of a boundary-layer on a flat surface from a known origin. It would seem equally justifiable to compare a given h_x with that *at the same station* if there was no step, thus extending the length of the flat surface for boundary-layer growth from the known origin. This method of presentation (which is not new) would yield a high heat-transfer multiplication factor than does the conventional procedure, but it would at least be realistic in tending towards unity as the step height is reduced indefinitely. This is only true of current practice at the step position; otherwise the limiting value of h_x/h_l falls increasingly below unity with distance downstream.

The distributions of r_x in the bottom panels of Figs. 1 and 2 show steadily increasing values in the separated region with local maxima before re-attachment. After re-attachment the curves become flat and generally tend towards constant values with increasing distance downstream. As Re_l increases, r_x at a given station decreases, and only near re-attachment for $Re_l = 8{\cdot}63 \times 10^5$ is r_x approximately the same as for attached flow along a flat plate. Elsewhere, and for higher Re_l, it is always less, a feature common to studies of separated regions involving flow reversal.

Naysmith [9] refers to considerable variation in recovery factor in his experiments [2]. This is also true of the results in Figs. 1 and 2, but the steady increase from a low value in the separated region is the reverse of the trend reported, admittedly for turbulent separation, by Brinich [1], Thomann [4] and Gadd *et al.* [5], where minima occur at, or downstream of, the position of maximum heat transfer, followed by a slow increase. However, near the step, Gadd *et al.* [5] show two point values of r_x, which they query, which correspond to the rising characteristics of the present work.

Relatively low values of r_x in the separated region are, of course, largely due to their being based according to convention on M_l, which is less than M_d. In contrast, downstream of the normal or oblique shock, where M_s is (in the present experiments) less than M_l, the use of the latter as reference yields artificially high values of r_x. It would seem more realistic to base r_x on the Mach number at the flow station under consideration. In Figs. 1 and 2, this would bring r_x during separation into closer accord with values for attached flow on a flat plate; it would also emphasize the local peaks in r_x before re-attachment.

CONCLUSIONS

This paper presents measurements of heat transfer and temperature recovery factor in separated flow in the entry region of a supersonic parallel diffuser of rectangular cross-section. A backward-facing step is created by the sudden enlargement in flow area where the generating nozzle joins the diffuser. The pre-separation Reynolds number is increased by raising the nozzle inlet stagnation pressure.

In laminar separation, both the heat transfer and the recovery factor increase with distance downstream for given Re_l and M_l, and reach maximum values near or before re-attachment. For a given position in the separated region, an increase in Re_l at constant M_l reduces the recovery factor and increases the heat transfer, the latter disproportionately at the highest Re_l attained. This may well indicate the onset of transition to turbulence during separation.

Conditions downstream of re-attachment depend on whether the shock wave at re-attachment is normal, at low Re_l, or oblique, at high Re_l. For a given Re_l the heat transfer always diminishes with downstream distance, but away from re-attachment after laminar separation it is greater when the flow is subsonic than when it is supersonic, despite the then higher Re_l. Recovery factors are also less behind the oblique shock system, but tend to become constant at great distances downstream.

While not universally true, the convention whereby heat-transfer coefficients and recovery factors behind the step are referred to, or based on, pre-separation conditions is, in the present configuration, generally less satisfactory than one based on conditions at the flow station to which the parameters refer, and, in the case of the heat-transfer coefficient, in the absence of a step.

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Résumé—On expose les résultats des mesures de transport de chaleur au début de la région d'élargissement d'un diffuseur supersonique à parois parallèles, région dans laquelle il se produit un décollement. On observe un transport de chaleur très élevé près du recollement lorsqu'on a un décollement que l'on pense accompagné de la transition. Après le recollement qui suit un décollement laminaire, le transport de chaleur est moindre avec un écoulement supersonique qu'avec l'écoulement subsonique provenant d'un choc normal au point de recollement.

Zusammenfassung—Messungen des Wärmeübergangs werden beschrieben für den Bereich der Eintrittserweiterung eines Überschallparalleldiffusors mit Strömungsablösung. Sehr hoher Wärmeübergang wird beim Wiederan legen mit angenommener Übergangsablösung beobachtet. Nach dem einer laminaren Ablösung folgenden Wiederanlegen ist der Wärmeübergang in Überschallströmung geringer als in einer Unterschallströmung, die sich aus einem normalen Stoss beim Wiederanlegen ergibt.